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DEFORMATION OF A LIQUID SURFACE BY AN IMPINGING GAS JET

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Abstract. The deformation of a liquid surface due to an impinging two-dimensional jet is considered assuming potential flow. This problem was solved numerically for small values of the Froude number λ by Olmstead and Raynor (1964). By using a different numerical procedure we solve the problem for larger values of λ up to $\lambda_1 \approx 6$. We show that the profile of the liquid surface contains a train of waves far away from the impinging jet.

1. Introduction. In an interesting theoretical and numerical work, Olmstead and Raynor (1964) examined the impact of a vertical jet on a stationary horizontal liquid surface (see Fig. 1). They derived the integro-differential system (1.1)–(1.3) below for the complex velocity $\xi(r)$ on the free surface of the liquid. They simplified it by replacing $\sin \theta$ by θ in (1.2) and then solved the resulting system numerically. Their numerical scheme converges, but only with few mesh points and for relatively small values of λ , with indication of a rapid growth in the error as λ increases.

We shall derive a numerical scheme different from theirs to solve (1.1)–(1.3) (see § 2). It enables us to compute the solution for arbitrary values of λ up to $\lambda = \lambda_1 \approx 6$. Our computations show that the liquid surface contains a train of waves far from the jet (see Figs. 1 and 2). These results are discussed in § 3.

The problem is characterized by the velocity V_j and the width b of the jet at $Y = \infty$, the gas density ρ_g , the liquid density ρ_l and the acceleration of gravity g . Taking b/π as the unit length and V_j as the unit velocity, Olmstead and Raynor (1964) derived the following nonlinear integro-differential equation for the complex velocity ξ on the free surface CJ_2 :

$$(1.1) \quad (1-r)^{-1/2} \pi(r) = -\frac{1}{\pi} \int_{-1}^{+1} \frac{(1-s)^{-1/2} \theta(s)}{s-r} ds \quad \text{on } CJ_2,$$

$$(1.2) \quad \sin \theta(r) = \frac{\pi \lambda}{3} (1-r) \frac{d}{dr} \left\{ \left[\frac{2^{1/2} - (1-r)^{1/2}}{2^{1/2} + (1-r)^{1/2}} \right]^{3/2} e^{3\pi i r} \right\} \quad \text{on } CJ_2,$$

$$(1.3) \quad \xi(r) = -\frac{(1+r)^{1/2}}{2^{1/2} + (1-r)^{1/2}} e^{3\pi i r} \quad \text{on } CJ_2.$$

Here λ is the Froude number defined by

$$(1.4) \quad \lambda = 2\rho_g V_j^2 / \rho_l g b.$$

The variable r in (1.1)–(1.3) is related to the potential function ϕ on CJ_2 by the formula

$$(1.5) \quad e^{-2\phi} = (1-r)/2, \quad r \in [-1, +1].$$

2. Numerical procedure. To solve (1.1)–(1.3) Olmstead and Raynor (1964) introduced the N mesh points r_l defined by

$$(2.1) \quad r_l = 1 - 2\left(\frac{l}{N}\right)^4, \quad l = 1, \dots, N.$$

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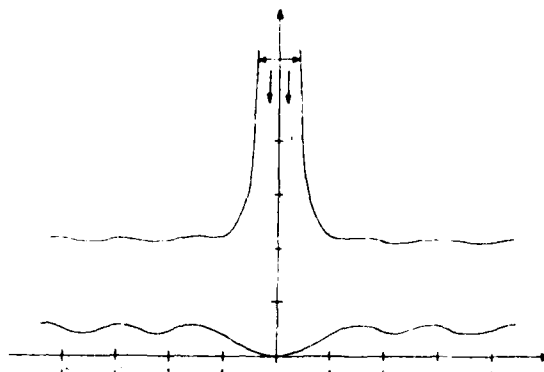


FIG. 1. Profile of the jet and the liquid surface for $\lambda = 0.6$. The axes and a sketch of the flow are also shown.

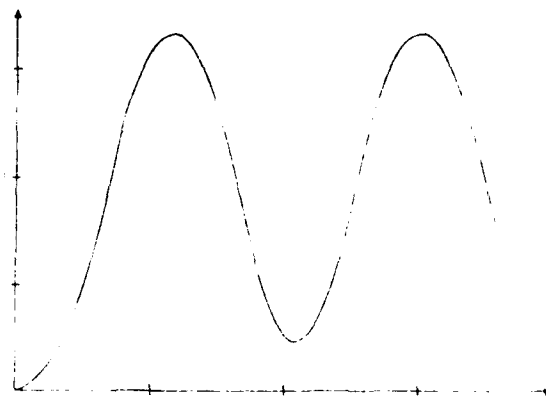


FIG. 2. Profile of the liquid surface for $\lambda = 5$.

Approximating $\sin \theta$ by θ in (1.2) and discretizing (1.1) and (1.2), they obtained a system of N equations with N unknowns $\tau_I = \tau(r_I)$, $I = 1, \dots, N$. This system was solved by Newton's method, using the solution corresponding to $\lambda = 0$ as the initial guess. Converged solutions of the algebraic system of equations for $N = 20$ and $\lambda \leq 1$ were presented in their paper. In addition, they gave an estimate of the error based on calculation of the vertical force on the liquid surface. It showed that the error grows rapidly with λ , reaching 8% for $\lambda = 1$. A small part of this error can be attributed to the approximation $\sin \theta \sim \theta$. However, the main error in their solutions comes from the fact that their liquid profiles do not contain waves. Numerical experiments show that Olmstead and Raynor's numerical procedures always give solutions of the discretized equations without waves. However, for any $\lambda > 0$, there exists a critical number of mesh points $N_c(\lambda)$ such that the scheme converges for $N < N_c(\lambda)$ and diverges for $N > N_c(\lambda)$. In addition the function $N_c(\lambda)$ decreases as λ increases.

Equations (1.1)–(1.3) are reminiscent of the integro-differential equation derived by Vanden-Broeck and Tuck (1977) and Vanden-Broeck, Schwartz and Tuck (1978) to describe the flow behind a moving body. Using the analogy between these two

problems and following the general philosophy of Vanden-Broeck, Schwartz and Tuck's method, the following successful numerical scheme was derived.

First we use (1.5) to rewrite (1.1)–(1.3) in terms of the new independent variable ϕ . Next we introduce the N mesh points ϕ_I given by

$$(2.2) \quad \phi_I = (I-1)E, \quad I = 1, \dots, N.$$

Here E is the interval of discretization. We also introduce the N corresponding unknowns,

$$(2.3) \quad \theta_I = \theta[r(\phi_I)], \quad I = 1, \dots, N.$$

By symmetry $\theta_1 = 0$, so only $N-1$ of the θ_I are unknown. We shall also use the $N-1$ intermediate mesh points $\phi_{I+1/2}$ given by

$$(2.4) \quad \phi_{I+1/2} = \frac{1}{2}(\phi_I + \phi_{I+1}), \quad I = 1, \dots, N-1.$$

We now compute

$$\tau_{I+1/2} = \tau[r(\phi_{I+1/2})]$$

in terms of θ_I by applying the trapezoidal rule to the integral in (1.1) rewritten with the new variable, with the mesh points ϕ_I . The symmetry of the discretization enables us to compute the Cauchy principal value as if it were an ordinary integral. The error inherent in approximating the integral by an integral over a finite interval was found to be negligible for NE large enough. Then from $\tau_{I+1/2}$ we compute $\tau_\phi(\phi_{I+1/2})$ and from θ_I we compute $\theta_{I+1/2}$. In the discretization we have used five-point difference formulas and four-point interpolation formulas and obtain the results in terms of θ_I .

Next we substitute into (1.2) the expressions so obtained for $\tau_{I+1/2}$, $\tau_\phi(\phi_{I+1/2})$ and $\theta_{I+1/2}$ at the $N-2$ points $\phi_{I+1/2}$, $I = 2, \dots, N-1$. Thus we obtain a system of $N-2$ nonlinear algebraic equations involving the $N-1$ unknowns θ_I , $I = 2, \dots, N$. The last equation is obtained by expressing θ_2 in terms of θ_3 , θ_4 and θ_5 by a three-point Lagrange extrapolation formula.

For each value of λ , the $N-1$ equations are solved by Newton's method with $\theta_I = 0$ as the initial guess. The remaining part of the computations follows closely Olmstead and Raynor's work. The scheme was found to be rapidly convergent and a solution of the algebraic equations with an error less than 10^{-10} was obtained in a few iterations. For each value of λ and NE , the value of E was decreased, and correspondingly N was increased, until the computed profiles remained unchanged, at least to graphical accuracy. The procedure was then repeated for large values of NE until the results became independent of NE . Most of the results presented in the next section were obtained with $E = 0.3$ and $N = 100$.

3. Discussion of the results. Typical profiles of the liquid surface for $\lambda = 0.6$ and $\lambda = 5$ are presented in Figures 1 and 2. In both figures the horizontal scale has been shrunk to show clearly the train of uniform waves far from the jet. It is convenient to characterize these waves by their steepness s defined as

$$(3.1) \quad s = \frac{\Delta y}{L}.$$

Here Δy is the difference of ordinate between a crest and a trough and L is the wavelength. The values of s are presented as a function of λ in Figure 3. As λ tends to zero, the steepness s tends to zero and the waves can be approximated by the sine waves of linear theory (see Fig. 1). It can easily be verified that the wavelength L of

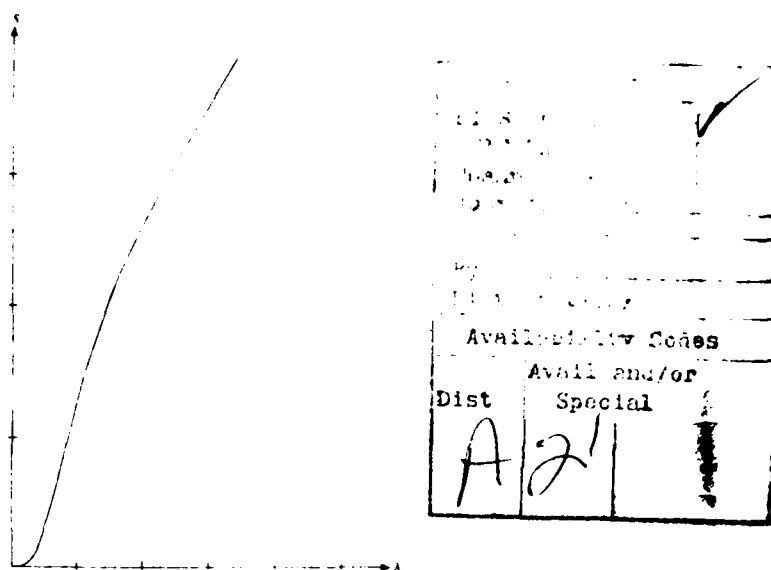


FIG. 3. Values of the steepness s as a function of the Froude number λ .

the waves satisfies the dispersion relation

$$(3.2) \quad L = \lambda \pi^2 \tanh \frac{\pi^2}{L} \quad \text{as } \lambda \rightarrow 0.$$

As λ increases, the steepness s increases and the waves start to develop sharp troughs and broad crests (see Fig. 2). These waves behave qualitatively like gravity surface waves of finite amplitude turned upside-down. This could be expected since the complete problem can be interpreted as a negative-gravity water-wave problem turned upside-down (Tuck (1975)). For steep waves the velocity is small at the troughs and large at the crests. Since the mesh points correspond to equal increments in the velocity potential, they are quite dense near the crests and sparse in the vicinity of the troughs. This nonuniform spacing of the mesh points for λ large limits the accuracy of the present numerical scheme. Similar difficulties were encountered before by Schwartz and Vanden-Broeck (1979) and Chen and Saffman (1980). Accurate solutions for $\lambda > \lambda_c$ could not be computed even with $N = 120$.

REFERENCES

- B. CHEN AND P. G. SAFFMAN (1980), *Steady gravity-capillary waves on deep water, II. Numerical results for finite amplitude*, Stud. Appl. Math., 62, pp. 95-111.
- W. E. OLMSTEAD AND S. RAYNOR (1964), *Depression of an infinite liquid surface by an incompressible gas jet*, J. Fluid Mech., 19, pp. 561-576.
- L. W. SCHWARTZ AND J.-M. VANDEN-BROECK (1979), *Numerical solution of the exact equations for capillary gravity waves*, J. Fluid Mech., 95, pp. 119-139.
- E. O. TUCK (1975), *On air flow over free surfaces of stationary water*, J. Austral. Math. Soc., Ser. B, 19, pp. 66-80.
- J.-M. VANDEN-BROECK, L. W. SCHWARTZ AND E. O. TUCK (1978), *Divergent low-Froude number series expansion of nonlinear free-surface flow problems*, Proc. Roy. Soc. Lond., A361, pp. 207-224.
- J.-M. VANDEN-BROECK AND E. O. TUCK (1977), *Computation of near bow or stern flows using series expansion in the Froude number*, Proc. 2nd International Conference on Numerical Ship Hydrodynamics, Berkeley, CA, pp. 371-381.

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